

Limits on laser wakefield accelerators

B. Yedierler^{a)} and S. Bilikmen

Department of Physics, Middle East Technical University, Inonu Blv. 06531 Ankara, Turkey

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The longitudinal and radial wakefields produced by a single laser pulse in a plasma are calculated. The limits on the laser wakefield acceleration because of diffraction, optical guiding, and energy loss due to radiation are examined. In particular for a bi-Gaussian laser beam, the energy gain about 4.6 GeV/cm s is estimated. A general constraint on the plasma density is presented. All the limits are compared and a localized density channel of width 4.6×10^{-5} cm is proposed. © 1999 American Institute of Physics. [S0034-6748(99)03304-3]

I. INTRODUCTION

The plasma particle accelerators has received a considerable interest in the past decade. Improvements on the laser technology and laboratory facilities made the plasma particle accelerators a strong alternative for the huge high-energy colliders. One sort of the plasma particle accelerators is called as the laser wakefield accelerators (LWFA), in which a short, intense laser pulse is used to create a wake in the plasma. Current systems of high power laser pulses produce a plasma wave which can accelerate electrons to the energies of one billion electron volts in a distance less than a centimeter.

When a short, intense laser pulse is sent to plasma, it generates a plasma wave whose amplitude is larger than that of the wave produced by the single laser pulse using the same total energy. Another series of pulses is used to push extra electrons into the wave, which accelerates them into a narrowly focused beam at nearly the speed of light. However, the problem is basically interaction of electromagnetic wave with particles, properties of the medium in which the particles present has great importance. The limits on the LWFA due to properties of the plasma are discussed.

II. CALCULATION OF THE WAKEFIELDS

The plasma fluctuations while the laser pulse propagates inside are described by the fluid equations. Assuming that the plasma is cold, and there is no background magnetic field, the motion of electrons, taking the background ions are stationary, is described by

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} + \nabla P = -e \left[\mathbf{E} + \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right) \right], \quad (1)$$

where \mathbf{v} and n are the velocity and the density of the electrons, respectively, P is the pressure. Furthermore, the equation of continuity and the Poisson's equation are employed.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{v}) = 0, \quad (2)$$

$$\nabla \cdot \mathbf{E} = 4 \pi \rho. \quad (3)$$

After defining the momentum \mathbf{p} , as the product of mass, velocity, and the number density, linearization of the velocity and density together with taking the ponderomotive force into consideration yields

$$m \frac{\partial \mathbf{v}}{\partial t} + m v_{ei} \mathbf{v} = -e (\nabla \phi + \nabla \phi_{NL}) \quad (4)$$

substituting the Poisson's equation into Eq. (4), a second-order partial differential equation for the density is obtained:

$$\frac{\partial^2 n}{\partial t^2} + v_{ei} \frac{\partial n}{\partial t} + n \omega_p^2 = -\frac{\omega_p^2}{4 \pi e} \nabla^2 \phi_{NL}, \quad (5)$$

where ω_p is the plasma frequency, v_{ei} is the electron ion collision frequency, and ϕ_{NL} is the nonlinear potential for the ponderomotive force of the laser beam. The Poisson's equation is used once more to define the number density in terms of the electrostatic potential. Having introduced this fact, the fluctuations in the plasma will be totally described by an inhomogeneous differential equation for the electrostatic potential.

$$\frac{\partial^2 \phi}{\partial t^2} + v_{ei} \frac{\partial \phi}{\partial t} + \phi \omega_p^2 = -\omega_p^2 \phi_{NL}. \quad (6)$$

The nonlinear potential is well defined in terms of the normalized vector potential \mathbf{a} such as $\phi_{NL} = -(mc^2/2e) |\mathbf{a}^2(r, z, t)|$ (this equation has appeared in Ref. 1 without an electron charge in the denominator, but in order to satisfy the unit of the potential, an electron charge e is needed there). Assuming the laser beam profile as bi-Gaussian and reducing the degrees of freedom of the problem by combining the time dependency and the longitudinal displacement together such that defining a time-dependent displacement $\xi = z - v_p t$, where v_p is the phase velocity of the excited plasma wave the nonlinear potential is then described as

$$\phi_{NL} = -\frac{mc^2}{2e} a_0^2 e^{-(2r^2/\sigma_r^2) - (\xi^2/\sigma_z^2)}.$$

Here the parameters σ_z and σ_r are *rms* pulse length and spot size, respectively. Thus, Eq. (6) turns out to be

^{a)}Electronic mail: byedier@newton.physics.metu.edu.tr

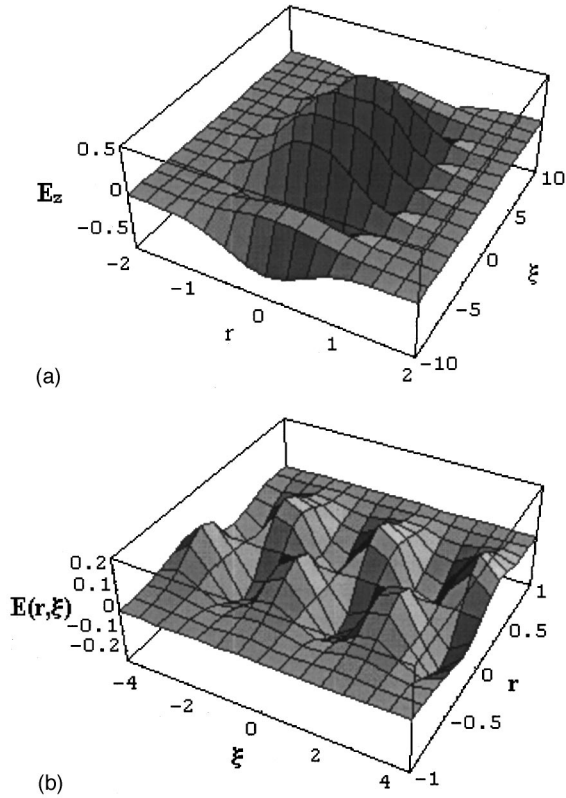


FIG. 1. Axial and the radial wakefields calculated for $a_0=1$ and $k_p\sigma_z=2$.

$$\begin{aligned} \frac{\partial^2 \phi(r, \xi)}{\partial y^2} + 2\alpha \frac{\partial \phi(r, \xi)}{\partial \xi} + k_p^2 \phi(r, \xi) \\ = -\frac{mk_p^2 c^2}{2e} a_0^2 e^{-((2r^2/\sigma_r^2) + (\xi^2/\sigma_z^2))}, \end{aligned} \quad (7)$$

where $\alpha = v_{ei}/2v_p$, and $k_p = \omega_p/v_p$. Solving Eq. (7) with $k^2 \equiv k_p^2 - \alpha^2$, yields

$$\begin{aligned} \phi(r, \xi) = -\frac{mc^2 k_p^2}{2ek} a_0^2 \\ \times \int_{\xi}^{\infty} d\xi' e^{\alpha(\xi' - \xi)} e^{-((2r^2/\sigma_r^2) + (\xi'^2/\sigma_z^2))} \\ \times \sin k(\xi' - \xi). \end{aligned} \quad (8)$$

The partial derivatives of Eq. (8) describes the so-called wakefields along the longitudinal and the radial directions. Making use of the error functions and the limits $\xi \rightarrow -\infty$,² $\xi \ll -\sigma_z$,¹ these wakefields are expressed as

$$\mathbf{E}_z(r, \xi) = \frac{\sqrt{\pi}}{4e} mc^2 \sigma_z k_p^2 e^{-(k^2 \sigma_z^2/4) - (r^2 \sigma_r^2)} \cos k\xi, \quad (9)$$

$$\mathbf{E}_r(r, \xi) = -\frac{\sqrt{\pi} mc^2}{e} a_0^2 \frac{r}{\sigma_r^2} \sigma_z e^{-(2r^2/\sigma_r^2) + (k_p^2 \sigma_z^2/4)} \sin(k\xi). \quad (10)$$

As schematically shown in Fig. 1, the axial wakefield reaches the maximum amplitude at $\xi = (n\pi/k)$ ($n = 0, 1, 2, \dots$). Naturally, these extremes alternate depending on the integer n , such that \mathbf{E}_z has a maximum for even n and it

has a minimum for odd n on the z axis. More clearly, it has a maximum at, $z = v_p t$, $(2\pi/k) + v_p t$, $(4\pi/k) + v_p t, \dots$ and it has a minimum at points, $z = (\pi/k) + v_p t$, $(3\pi/k) + v_p t, \dots$

III. DIFFRACTION LIMITED LWFA

Because of the fact that there is a remarkable refractive index for the plasma media, it is diffracted when the laser beam is sent through the plasma. There is a restriction on the interaction distance due to diffraction. In order to particle acceleration to take place, the beam must not spread. The physical phenomenon is explained by the concept of Rayleigh length. The acceleration takes place within the Rayleigh length, described as, $\pi Z_R = (\pi^2 R_0^2)/\lambda_0$, where R_0 is the focal spot radius and λ_0 is the laser wavelength.

Since the laser beam accelerates the electrons up to the Rayleigh length, the energy gain of the electrons is given by

$$\Delta W = \frac{\sqrt{\pi}}{k} mc^2 \sigma_z k_p^2 e^{-(k^2 \sigma_z^2/4) - (r^2/\sigma_r^2)} \sin k(\pi Z_R - v_p t). \quad (11)$$

Expressing the focal spot radius in terms of the laser wavelength and the phase velocity of the laser beam, the energy gain becomes proportional to $\sin(n\pi)$. Therefore; the particles do not gain energy where \mathbf{E}_z has a minimum. For some typical parameters for a Ti: sapphire T^3 laser, of 780 nm wavelength and $\sim 10 \mu\text{m}$ focal spot radius together with plasma wavelength of $\sim 68 \mu\text{m}$,¹ the energy gain within the Rayleigh length is estimated as $\sim 4.6 \text{ GeV/cm s}$.

IV. OPTICALLY GUIDED LWFA

Arrangement of the optical properties of the plasma in order to increase the energy gain, so-called optical guiding, is mostly established by refractive guiding. In terms of the density profile, $n(r)$, of the plasma the refractive index, $\eta(r)$, is given by

$$\eta(r) = 1 - \frac{\omega_p^2}{2\omega_0^2} \frac{n(r)}{n_0} \frac{1}{1 + \phi_s}, \quad (12)$$

where ω_0 and ω_p are the laser and the plasma frequencies, respectively and ϕ_s is the slow part of the potential. Since the term $1 + \phi_s$ is explicitly stated as³ $1 + \phi_s = \sqrt{1 + (|a|^2/2)}$, the index of refraction of the plasma for a bi-Gaussian laser beam is expressed by

$$\eta(r) = 1 - \frac{\omega_p^2}{\omega_0^2} \frac{n(r)}{n_0} \left[1 + \frac{1}{2} a_0^2 e^{-(r^2/\sigma_r^2) - (\xi^2/\sigma_z^2)} \right]^{-1/2}. \quad (13)$$

It is known that the optical guiding is possible when the index of refraction exhibits a maximum on axis,⁴ derivative of Eq. (13) with respect to r yields a first-order ordinary differential equation. Integration of this equation shows that

$$n(r) < \sqrt{1 + \frac{1}{2} a_0^2 e^{-(r^2/\sigma_r^2)}}. \quad (14)$$

Equation (14) explains that the plasma should be arranged in such a way that the density profile should be less than the square root of unity plus half of the normalized vector potential.

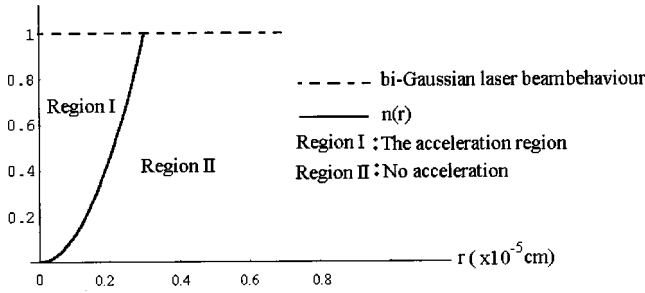


FIG. 2. Comparison of the bi-Gaussian packet with the density profile.

Considering a bi-Gaussian beam, assuming a parabolic plasma density, $n(r) = \Delta n(r^2/R_0^2)$ and assuming $\Delta n \sim 1.1 \times 10^{18} \text{ cm}^{-3}$, and $\sim 10 \mu\text{m}$ focal spot radius,¹ a numerical value for the channel width can be estimated by using Eq. (14). Since the second term in the square root vanishes exponentially, the right-hand side of Eq. (14) shows a constant behavior. However, the parabolic density increases with increase in the radial coordinate. Then this inequality is solved graphically as shown in Fig. 2, and the channel width is estimated as, $\sim 4.6 \times 10^{-5} \text{ cm}$.

The relationship between the index of refraction and density profile describes a density channel. The density channel in other words is well described by Eq. (14). The solution of the inequality gives an upper limit for the density channel. Therefore, it has been seen that the refractive guiding automatically satisfies another guiding mechanism called as the channel guiding. Hence, the limit due to channel guiding, namely the phase detuning distance must be handled within those restrictions. Using the same parameters mentioned above, the length along which the electrons stay in phase with the laser beam, L_ϕ , which is given by $L_\phi = \lambda_p(\lambda_p/\lambda_0)^2$, has a numerical value, $L_\phi \sim 5.93 \text{ mm}$.

V. ENERGY LOSS

While the laser beam propagates in the plasma, it loses some of its original energy by radiation. A characteristic time scale during which the packet energy is altered by the emission of plasma wave is obtained from the ratio of the initial packet energy to the change in the energy per unit time. Therefore, taking the initial packet energy⁵ as

$$W_0 = \frac{m^2 \omega^2}{8 \pi e^2} a_0^2 \sigma_z e^{-(2r^2/\sigma_r^2)} \sqrt{\frac{\pi \sigma_z}{\xi}} \quad (15)$$

and the change of it per unit time into consideration:

$$\delta W = \frac{m n_0 k_p^2}{32 v_g} \left[\left(\int_{-\infty}^{\infty} |\mathbf{a}(\xi)|^2 \cos k_p \xi d\xi \right)^2 + \left(\int_{-\infty}^{\infty} |\mathbf{a}(\xi)|^2 \cos k_p \xi d\xi \right)^2 \right], \quad (16)$$

the characteristic time is calculated as

$$T_{\text{rad}} = \frac{W_0}{\delta W} = \frac{4 \omega^2}{k_p a_0^2} \sqrt{(\xi^2 + 1)} \left(\frac{\lambda_p}{\xi} \right) e^{(2r^2/\sigma_r^2) + 2}. \quad (17)$$

It must be noted that the distance traversed by the packet before its energy is decreased by the radiation is approximated as $T_{\text{rad}} v_g \cong T_{\text{rad}} c$.⁶ Then the limit due to the energy loss, the energy depletion distance is expressed as

$$L_d = \frac{4 c \omega_0}{a_0^2 k_p^2} \sqrt{(\xi^2 + 1)} \left(1 - \frac{4 \pi^2}{\lambda_p^2} \right) \left(\frac{\lambda_p}{\xi} \right). \quad (18)$$

VI. DISCUSSION

The laser wakefield acceleration scheme needs detailed restrictions on the interaction length and the band, in which the electrons are accelerated on the wakefields presented in Fig. 1. Those restrictions are all results from the guiding mechanisms that are intended to increase the energy gain by either arranging the plasma or the laser parameters.

Having concentrated on all the schemes separately, it has been seen that each paradigm results a limit on the whole scheme. Although, the Rayleigh length, phase detuning distance, and the energy depletion distance have completely different origins, since all of them causes a limit on either the longitudinal or the radial displacements, there is a significant correlation between them. Therefore, combining of all the results one can conclude that before the energy of the initial laser beam reduces to half of its original value, the electrons must be stay in phase with the beam only in the extend of the Rayleigh length.

Behavior of the wakefields, on the other hand, shows that the energy gain is not a continuous event. Therefore, together with the guiding mechanisms a serious increment on the energy gain can be satisfied by also maximizing the number of peaks within the Rayleigh length. This is nothing but to modulate the laser packet as compact as possible.

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